

MASSIVE SUPERPARTICLE WITH TENSORIAL CENTRAL CHARGES

S. Fedoruk¹ and V. G. Zima²

¹ *Ukrainian Engineering–Pedagogical Academy,
61003 Kharkiv, 16 Universitetska Str., Ukraine*

e-mail: fed@postmaster.co.uk

² *Kharkiv National University,
61077 Kharkiv, 4 Svobody Sq., Ukraine*

e-mail: zima@postmaster.co.uk

Abstract

We construct the manifestly Lorenz-invariant formulation of the $N = 1$ $D = 4$ massive superparticle with tensorial central charges. The model contains a real parameter k and at $k \neq 0$ possesses one κ -symmetry while at $k = 0$ the number of κ -symmetry is two. The equivalence of the formulations at all $k \neq 0$ is obtained. The local transformations of κ -symmetry are written out. It is considered the using of index spinor for construction of the tensorial central charges. It is obtained the equivalence at classical level between the massive $D = 4$ superparticle with one κ -symmetry and the massive $D = 4$ spinning particle

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1 Introduction

Last time there is the great interest in the analysis of the supersymmetric models possessing supersymmetry with additional nonscalar central charges [1]-[8]. Although the tensorial central charges in the supersymmetry algebra are associated with topological contributions of the superbrane theories it is attractive to obtain the superparticle models having this symmetry. Recently such supersymmetric particle models were obtained in massless case [9], for $D = 4$ with two or three local κ -symmetries.

In present work we construct the model of the massive $N = 1$ $D = 4$ superparticle with tensorial central charges possessing one or two local κ -symmetries ¹. In such a way we obtain in usual space-time dimensions $D = 4$ the superparticle with a single κ -symmetry which is equivalent physically to the usual spinning (spin 1/2) particle [11, 12] in the positive energy sector.

In the pseudoclassical approach the Lagrangian of spinning particle has the following form [11, 12]

$$L_{1/2} = p^\mu \dot{x}_\mu + \frac{i}{2}(\psi^\mu \dot{\psi}_\mu + \psi_5 \dot{\psi}_5) - \frac{e}{2}(p^2 + m^2) - i\chi(p\psi + m\psi_5). \quad (1)$$

The spin variables in this description are the Grassmannian (pseudo)vector ψ_μ and the Grassmannian (pseudo)scalar ψ_5 . Besides mass constraint $T \equiv p^2 + m^2 \approx 0$ in Hamiltonian formalism the physical sector of the model is subjected to the Grassmannian constraints from which one Dirac constraint

$$D \equiv p^\mu \psi_\mu + m\psi_5 \approx 0 \quad (2)$$

plays the role of the first class constraint and five self-conjugacy condition for the Grassmannian variables

$$g^\mu \equiv p_\psi^\mu - \frac{i}{2}\psi^\mu \approx 0, \quad g_5 \equiv p_{\psi_5} - \frac{i}{2}\psi_5 \approx 0 \quad (3)$$

are the second class constraints. Thus the number of physical Grassmannian degrees of freedom in the model (1) is [number of $(\psi_\mu, \psi_5, p_{\psi_\mu}, p_{\psi_5})$] - [number of the second class constraints (g_μ, g_5)] - 2[number of the first class constraint (D)] = 3.

¹The Lagrangian of the massive superparticle with vector central charge and with two κ -symmetries has been presented already in [10]

The usual model of the massive CBS superparticle [13] with Grassmannian spinor coordinates θ^α , $\bar{\theta}^{\dot{\alpha}}$ has only the fermionic spinor constraints

$$d_{\theta\alpha} \equiv -ip_{\theta\alpha} - (\hat{p}\bar{\theta})_\alpha \approx 0, \quad \bar{d}_{\theta\dot{\alpha}} \equiv -i\bar{p}_{\theta\dot{\alpha}} - (\theta\hat{p})_{\dot{\alpha}} \approx 0$$

which all are the second class constraints. Here the number of the physical Grassmannian degrees of freedom is $[\text{number } (\theta^\alpha, \bar{\theta}^{\dot{\alpha}}, p_{\theta\alpha}, \bar{p}_{\theta\dot{\alpha}})] - [\text{number } (d_\theta, \bar{d}_\theta)] = 4$. In order to obtain desired three physical fermionic degrees of freedom it is necessary that from fermionic four spinor constraints three constraints are of the second class whereas one constraint should be of the first class. Such situation with nonsymmetric separation of the fermionic constraints into the ones of first and second class has been proposed in massless superparticle models [9] as well as in the massive particle case [14]. Precisely the situation with one first class fermionic constraint has been present in [14] in the construction of $N = 4 \rightarrow N = 1$ PBGS in $d = 1$. The relation between that model and our one will be given below. Thus in the massive case the equivalence of spinning particle and superparticle with tensorial central charges with one κ -symmetry is expected. Let us note that in massless case [15, 16] the spinning particle is equivalent, at least on classical level, to the usual CBS superparticle without any central charges. This fact of identifying the local fermionic invariances of spinning particle and κ -symmetries of superparticle is essential for superfield formulation of massless superparticle theory [15, 16] and consequent generalizations on superbranes [17].

Accounting above mentioned preliminary arguments for the possible relation between massive spinning particle and massive superparticle with tensorial central charges we take the following way for construction of the superparticle model. We shall realize the covariant transition, under preservation of the physical contents, from the model of the massive spinning particle to the system with Grassmannian spinor variables. As result of this procedure we arrive at model of the $N = 1$ $D = 4$ massive superparticle with tensorial central charges possessing one local fermionic invariance (κ -symmetry).

Covariant transition from the Grassmannian vector ψ_μ and scalar ψ_5 to the Grassmannian spinors $\theta^\alpha, \bar{\theta}^{\dot{\alpha}}$ requires using of the commuting spinor variables. We introduce dynamical commuting spinor

variables $\zeta^\alpha, \bar{\zeta}^{\dot{\alpha}} = \overline{(\zeta^\alpha)}$. Their canonical conjugate momenta are $v_\alpha, \bar{v}_{\dot{\alpha}}$. Introduced spinors are subject to the condition

$$r - j \equiv \zeta \hat{p} \bar{\zeta} - j \approx 0 \quad (r \equiv \zeta \hat{p} \bar{\zeta}). \quad (4)$$

This constraint inherent in the index spinor approach [18, 10, 19] gives us the completeness condition

$$r \delta_\alpha^\beta = \zeta_\alpha (\bar{\zeta} \tilde{p})^\beta + (\hat{p} \bar{\zeta})_\alpha \zeta^\beta$$

for spinors $\zeta, \hat{p} \zeta$. Here matrix \tilde{p} is the contraction of the space-time momentum and σ -matrices with upper spinor indices, $\tilde{p} \equiv p^\mu \tilde{\sigma}_\mu$, $\tilde{p} = (p^{\dot{\alpha}\alpha})$. Corresponding matrix with lower indices is denoted by \hat{p} , $\hat{p} = p^\mu \sigma_\mu = (p_{\alpha\dot{\alpha}})$. Numerical constant $j \neq 0$ plays the role of “classical spin” in the index spinor formalism [18, 10, 19]. We assume that the dynamics of the bosonic spinor variables is determined by the Lagrangian of the following form

$$L_{\text{b.s.}} = \dot{\zeta} v + \bar{v} \dot{\bar{\zeta}} - \lambda (\zeta \hat{p} \bar{\zeta} - j). \quad (5)$$

One can exclude the variable ζ using its equation of motion. Thus we obtain the second order Lagrangian

$$L_{\text{b.s.}}^{(2)} = \Lambda^{-1} \left[m^{-1} \dot{v} \hat{p} \dot{v} + \Lambda^2 (j/m) \right]$$

with $\Lambda \equiv m\lambda$. This Lagrangian describes the motion of a point in complex two-dimensional space parametrized by the Weyl spinor v . Canonically conjugate space parametrized by ζ is restricted by the constant (4) and is obviously isomorphic to the compact group manifold $SU(2)$. Formally, the constant j/m plays here the role of point “mass”.

The total system which we consider as initial under transition to Grassmannian spinors is in fact the sum of the two sectors coupled through the space-time coordinates. One of these sectors is the usual massive spinning particle with Lagrangian (1) whereas the second is the sector of the bosonic spinor with Lagrangian (5). Thus the Lagrangian of the initial system has the following form

$$L = L_{1/2} + L_{\text{b.s.}}$$

$$\begin{aligned}
&= p\dot{x} + \frac{i}{2}(\psi\dot{\psi} + \psi_5\dot{\psi}_5) - \frac{e}{2}(p^2 + m^2) - i\chi(p\psi + m\psi_5) \\
&\quad + \dot{\zeta}v + \bar{v}\dot{\bar{\zeta}} - \lambda(\zeta\hat{p}\bar{\zeta} - j).
\end{aligned} \tag{6}$$

As result of the constraint $\zeta\hat{p}\bar{\zeta} = j$ the sign of the constant j defines the sign of the energy. In following we consider the positive energy sector where $j > 0$.

In this paper we use the $D = 4$ spinor conventions of [20].

2 Massive superparticle with tensorial central charges. Lagrangian

The conversion of spinning particle model described by the Grassmannian variables ψ_μ, ψ_5 to the model with the Grassmannian spinor variables $\theta^\alpha, \bar{\theta}^{\dot{\alpha}}$ is realized by the general resolution [10] of the form

$$\psi_\mu = r^{-1/2}(\theta\sigma_\mu\tilde{p}\zeta + \bar{\zeta}\tilde{p}\sigma_\mu\bar{\theta}) - m\rho\zeta\sigma_\mu\bar{\zeta}, \tag{7}$$

$$\psi_5 = r^{-1/2}m(\zeta\theta + \bar{\theta}\bar{\zeta}) + r\rho + \tilde{\psi}_5. \tag{8}$$

The initial Grassmannian variables ψ_μ, ψ_5 (5 variables) are expressed in terms of two Grassmannian scalars $\rho, \tilde{\psi}_5$ and three components of spinor θ . Just for projections of $\psi_\mu \equiv -\frac{1}{2}\tilde{\sigma}_\mu^{\dot{\alpha}\alpha}\hat{\psi}_{\alpha\dot{\alpha}}$ in the basis formed by spinors $\zeta^\alpha, (\bar{\zeta}\tilde{p})^\alpha$ we have

$$\zeta\hat{\psi}\bar{\zeta} = 2r^{1/2}(\zeta\theta + \bar{\theta}\bar{\zeta}), \quad \bar{\zeta}\tilde{p}\hat{\psi}\tilde{p}\zeta = 2mr^2\rho, \tag{9}$$

$$\zeta\hat{\psi}\tilde{p}\zeta = 2r^{1/2}(\zeta\hat{p}\bar{\theta}), \quad \bar{\zeta}\tilde{p}\hat{\psi}\bar{\zeta} = 2r^{1/2}(\bar{\theta}\hat{p}\bar{\zeta}), \tag{10}$$

where $\hat{\psi} = \psi^\mu\sigma_\mu$. The fourth component of the spinor

$$\phi = i(\theta\zeta - \bar{\zeta}\bar{\theta}) \tag{11}$$

does not participate in the expression for ψ -variables. The inversion of (7), (8) and (11) looks as follows

$$\theta_\alpha = \frac{1}{4}r^{-3/2} \left[(\zeta\hat{\psi}\bar{\zeta})(\hat{p}\bar{\zeta})_\alpha + 2(\bar{\zeta}\tilde{p}\hat{\psi}\bar{\zeta})\zeta_\alpha \right] + \frac{i}{2}r^{-1}\phi(\hat{p}\bar{\zeta})_\alpha,$$

$$\bar{\theta}_{\dot{\alpha}} = \frac{1}{4}r^{-3/2} \left[(\zeta\hat{\psi}\bar{\zeta})(\zeta\hat{p})_{\dot{\alpha}} + 2(\zeta\tilde{\psi}\hat{p}\zeta)\bar{\zeta}_{\dot{\alpha}} \right] - \frac{i}{2}r^{-1}\phi(\zeta\hat{p})_{\dot{\alpha}},$$

$$\rho = \frac{1}{2m}r^{-2}(\bar{\zeta}\hat{p}\tilde{\psi}\hat{p}\zeta), \quad \tilde{\psi}_5 = \frac{1}{m}(p^\mu\psi_\mu + m\psi_5) - (2mr)^{-1}(\zeta\hat{\psi}\bar{\zeta})(p^2 + m^2).$$

In the new variables the Dirac constraint takes a simple form. On mass shell $p^2 + m^2 = 0$ we have

$$D = p\psi + m\psi_5 = m\tilde{\psi}_5 \approx 0. \quad (12)$$

Moreover, we can extract from the new variables a pure gauge degree of freedom for fermionic local symmetry of the spinning particle [11, 12] (world-line supersymmetry)

$$\delta\chi = \epsilon, \quad \delta e = -2i\epsilon\chi, \quad \delta\psi_\mu = -\epsilon p_\mu, \quad \delta\psi_5 = -\epsilon m, \quad \delta x_\mu = i\epsilon\psi_\mu.$$

In the new variables this transformation takes the form

$$\delta\theta_\alpha = -\frac{1}{4}\epsilon r^{-1/2}(\hat{p}\bar{\zeta})_\alpha, \quad \delta\bar{\theta}_{\dot{\alpha}} = -\frac{1}{4}\epsilon r^{-1/2}(\zeta\hat{p})_{\dot{\alpha}},$$

$$\delta\rho = -\frac{1}{2}\epsilon mr^{-1}, \quad \delta\tilde{\psi}_5 = -\frac{1}{2m}\epsilon(p^2 + m^2) \approx 0.$$

Thus, the only transformed are the variable ρ and one component of spinor θ

$$\delta(\theta\zeta + \bar{\zeta}\bar{\theta}) = \frac{1}{2}\epsilon r^{1/2}.$$

Subsequently the combination $\rho + mr^{-3/2}(\theta\zeta + \bar{\zeta}\bar{\theta})$ of this component θ and ρ is invariant under the gauge transformations, $\delta[\rho + mr^{-3/2}(\theta\zeta + \bar{\zeta}\bar{\theta})] = 0$, whereas the variable

$$\rho - mr^{-3/2}(\theta\zeta + \bar{\zeta}\bar{\theta}) \quad (13)$$

is the pure gauge degree of freedom, $\delta[\rho - mr^{-3/2}(\theta\zeta + \bar{\zeta}\bar{\theta})] = -mr^{-1}\epsilon$.

Accounting the equation of motion for bosonic spinor $\dot{\zeta} = 0$ and substituting the resolving expressions (7), (8) for ψ_μ , ψ_5 in the Lagrangian (6) we arrive at the Lagrangian

$$L = p(\dot{x} - i\dot{\theta}\sigma\bar{\theta} + i\dot{\theta}\sigma\bar{\dot{\theta}}) - im^2r^{-1}(\theta\zeta\bar{\zeta}\bar{\dot{\theta}} - \dot{\theta}\zeta\bar{\zeta}\bar{\theta})$$

$$+ \frac{i}{2}r^2 \left[\rho + mr^{-3/2}(\theta\zeta + \bar{\zeta}\bar{\theta}) \right] \left[\dot{\rho} + mr^{-3/2}(\dot{\theta}\zeta + \bar{\zeta}\bar{\dot{\theta}}) \right]$$

$$\begin{aligned}
& + \frac{i}{2}r \left[\rho - mr^{-3/2}(\theta\zeta + \bar{\zeta}\bar{\theta}) \right] \dot{\tilde{\psi}}_5 + \frac{i}{2}r\tilde{\psi}_5 \left[\dot{\rho} - mr^{-3/2}(\dot{\theta}\zeta + \dot{\bar{\zeta}}\bar{\theta}) \right] \\
& + \frac{i}{2}\tilde{\psi}_5\dot{\tilde{\psi}}_5 - im\chi\tilde{\psi}_5 - \frac{e}{2}(p^2 + m^2) \\
& + \dot{\zeta}v + \bar{v}\dot{\bar{\zeta}} - \lambda(\zeta\hat{p}\bar{\zeta} - j).
\end{aligned} \tag{14}$$

It should be stressed that the equation $\dot{\zeta} = 0$ for bosonic spinor, which has been used for derivation of the Lagrangian (14), is reproduced by the same Lagrangian (14). As we see from the Lagrangian, the gauge variable (13) $\rho - mr^{-3/2}(\theta\zeta + \bar{\zeta}\bar{\theta})$ is the corresponding conjugate variable for $\tilde{\psi}_5$ which generates the local transformations. The simpler gauge fixing condition for it

$$\rho - mr^{-3/2}(\theta\zeta + \bar{\zeta}\bar{\theta}) = 0$$

gives us the possibility to resolve the scalar ρ in term of spinor projection $(\theta\zeta + \bar{\zeta}\bar{\theta})$. We take the more general condition of this type

$$\rho - mr^{-3/2}(\theta\zeta + \bar{\zeta}\bar{\theta}) = 2(k-1)mr^{-3/2}(\theta\zeta + \bar{\zeta}\bar{\theta}) \tag{15}$$

which is the gauge fixing condition at all k except $k = 0$. At $k = 0$ (15) is reduced to the condition on gauge invariant variable

$$\rho + mr^{-3/2}(\theta\zeta + \bar{\zeta}\bar{\theta}) = 0$$

and of course it is not a gauge fixing.

Substituting in the Lagrangian (14) the constraint condition $\tilde{\psi}_5 = 0$ (the equation of motion for the Lagrange multiplier χ) and the expression

$$\rho = (2k-1)mr^{-3/2}(\theta\zeta + \bar{\zeta}\bar{\theta}) \tag{16}$$

(following from the gauge fixing condition (15)) we obtain the Lagrangian

$$\begin{aligned}
L = & p\dot{\omega}_\theta + iZ_{\alpha\beta}\theta^\alpha\dot{\theta}^\beta + i\bar{Z}_{\dot{\alpha}\dot{\beta}}\bar{\theta}^{\dot{\alpha}}\dot{\bar{\theta}}^{\dot{\beta}} + iZ_{\alpha\dot{\beta}}(\theta^\alpha\dot{\bar{\theta}}^{\dot{\beta}} - \dot{\theta}^\alpha\bar{\theta}^{\dot{\beta}}) - \frac{e}{2}(p^2 + m^2) \\
& + \dot{\zeta}v + \bar{v}\dot{\bar{\zeta}} - \lambda(\zeta\hat{p}\bar{\zeta} - j).
\end{aligned} \tag{17}$$

In this expression $\omega_\theta \equiv \dot{\omega}_\theta d\tau = dx - id\theta\sigma\bar{\theta} + i\theta\sigma d\bar{\theta}$ is the usual $N = 1$ superinvariant ω -form. The quantities $Z_{\alpha\beta} = Z_{\beta\alpha}$, $\bar{Z}_{\dot{\alpha}\dot{\beta}} = \overline{(Z_{\alpha\beta})}$ and $Z_{\alpha\dot{\beta}} = \overline{(Z_{\beta\dot{\alpha}})}$ are expressed in terms of bosonic spinor ζ (for similar formula see [9])

$$Z_{\alpha\beta} = 2k^2 m^2 j^{-1} \zeta_\alpha \zeta_\beta, \quad Z_{\alpha\dot{\beta}} = (2k^2 - 1) m^2 j^{-1} \zeta_\alpha \bar{\zeta}_{\dot{\beta}}. \quad (18)$$

$Z_{\alpha\beta}$ and $\bar{Z}_{\dot{\alpha}\dot{\beta}}$ are tensor central charges (types $(1,0)$ and $(0,1)$) and $Z_{\alpha\dot{\beta}}$ is vector one (type $(1/2, 1/2)$) for the $D = 4$ $N = 1$ supersymmetry algebra [1]-[8].

The same result is obtained if we consider the connection of the systems (6) and (17) in the Hamiltonian formalism. Precisely there is the canonical transformation which connect the models with each other. Now in order to make equal the number of Grassmannian variables in the models we introduce pure gauge variable ϕ in the initial model of the spinning particle. Its pure gauge nature is achieved by the presence of the first class constraint

$$p_\phi \approx 0 \quad (19)$$

in the initial model. So in the canonical transformation we imply that the term $p_\phi \dot{\phi} - \mu p_\phi$ is added to the Lagrangian (6). Here μ is Lagrange multiplier. The resolution of ϕ in terms of the spinors is given by the expression (11).

As the generating function of the canonical transformation from system with coordinates $\psi_\mu, \psi_5, \phi, x^\mu, \zeta^\alpha, \bar{\zeta}^{\dot{\alpha}}$ to the system with coordinates $\theta^\alpha, \bar{\theta}^{\dot{\alpha}}, \rho, \tilde{\psi}_5, x'^\mu, \zeta'^\alpha, \bar{\zeta}'^{\dot{\alpha}}$ we take

$$\begin{aligned} F = & -p'_\psi \psi_\mu(p_\mu, \zeta, \theta, \rho) - p_{\psi_5} \psi_5(\zeta, \theta, \rho, \tilde{\psi}_5) - p_\phi \phi(\zeta, \theta) \\ & + \zeta^\alpha v'_\alpha + \bar{v}'_{\dot{\alpha}} \bar{\zeta}^{\dot{\alpha}} - p^\mu x'_\mu. \end{aligned} \quad (20)$$

Here the expressions for old variables in term of new ones from the right hand side of the equations (7), (8), (11) have been used. That construction of the generating function (20) reproduces by definition of the canonical transformation the resolution (7), (8), (11) of the initial Grassmannian coordinates in spinors $\psi_\mu = -\partial_l F / \partial p^\mu_\psi$, $\psi_5 = -\partial_l F / \partial p_{\psi_5}$, $\phi = -\partial_l F / \partial p_\phi$ and leaves invariable bosonic spinor

coordinates $\zeta'^\alpha = \partial F / \partial v'_\alpha = \zeta^\alpha$, $\bar{\zeta}'^{\dot{\alpha}} = \partial F / \partial \bar{v}'_{\dot{\alpha}} = \bar{\zeta}^{\dot{\alpha}}$ and the momentum vector $p'_\mu = -\partial F / \partial x'^\mu = p_\mu$.

The expression of new Grassmannian momenta in terms of initial ones are

$$\begin{aligned} p_{\theta\alpha} &= -\partial_r F / \partial \theta^\alpha = r^{-1/2} (\sigma_\mu \tilde{p} \zeta)_\alpha p_\psi^\mu - m r^{-1/2} \zeta_\alpha p_{\psi 5} + i \zeta_\alpha p_\phi, \\ \bar{p}_{\theta\dot{\alpha}} &= -\partial_r F / \partial \bar{\theta}^{\dot{\alpha}} = r^{-1/2} (\bar{\zeta} \tilde{p} \sigma_\mu)_{\dot{\alpha}} p_\psi^\mu - m r^{-1/2} \bar{\zeta}_{\dot{\alpha}} p_{\psi 5} - i \bar{\zeta}_{\dot{\alpha}} p_\phi, \\ p_\rho &= -\partial_r F / \partial \rho = -m (\zeta \sigma_\mu \bar{\zeta}) p_\psi^\mu + r p_{\psi 5}, \quad p_{\tilde{\psi} 5} = -\partial_r F / \partial \tilde{\psi}_5 = p_{\psi 5}. \end{aligned}$$

The expressions of the initial bosonic spinor momenta $v_\alpha = \partial F / \partial \zeta^\alpha$, $\bar{v}_{\dot{\alpha}} = \partial F / \partial \bar{\zeta}^{\dot{\alpha}}$ and space-time coordinate $x_\mu = -\partial F / \partial p^\mu$ in terms of the new phase space coordinates contain besides corresponding new phase variables the additional terms depending on the new Grassmannian phase space variables. These terms arise because of the dependence of the resolution expressions (7), (8), (11) on ζ , $\bar{\zeta}$ and p . Here we do not need in the explicit form the expressions for v' , \bar{v}' and x' due to independence of all constraints on these phase variables.

Now we eliminate the variables $\tilde{\psi}_5$, $p_{\tilde{\psi} 5}$ by means of the Dirac constraint (2) and gauge fixing condition for Dirac constraint

$$p_{\tilde{\psi} 5} - i(k-1)mr^{-1/2} [\theta\zeta + \bar{\zeta}\bar{\theta}] \approx 0 \quad (21)$$

at $k \neq 0$ ². After fulfilment of the additional canonical transformation $p_\rho \rightarrow p_{\rho'} = p_\rho - i k m r^{1/2} [\theta\zeta + \bar{\zeta}\bar{\theta}]$, which leads to resolving form $p_{\rho'} \approx 0$ of one Fermi-constraint from (3), we eliminate the variables ρ , p_ρ with the help of two from five second class Fermi-constraints (3). Because of the resolving form of the constraints with respect to eliminated variables, $\tilde{\psi}_5 \approx 0$ and $p_{\rho'} \approx 0$, the Dirac brackets for remaining variables are the same as their Poisson brackets. After that the remaining Grassmannian constraints take the following form

$$\bar{\zeta} \tilde{p} p_\theta - \bar{p}_\theta \tilde{p} \zeta \approx 0, \quad (22)$$

²The diagonalized Dirac constraint $D' \equiv D - i p^\mu g_\mu - i m g_5 = -i[p_\mu(p_\psi^\mu + \frac{i}{2}\psi^\mu) + m(p_{\psi 5} + \frac{i}{2}\psi_5)] \approx 0$ has in new variables the form $D' = \frac{i}{4}r^{-1/2} [\bar{\zeta} \tilde{p} p_\theta + \bar{p}_\theta \tilde{p} \zeta] - \frac{i}{2}mr^{-1}p_\rho + \frac{1}{2}m\tilde{\psi}_5 \approx 0$. The Poisson bracket of the condition (21) and D' is equal to $(km)/2$, i.e. at $k=0$ the condition (21) does not fix the gauge for the Dirac constraint.

$$[\bar{\zeta}\tilde{p}p_\theta + \bar{p}_\theta\tilde{p}\zeta] - 4ik^2m^2[\theta\zeta + \bar{\zeta}\bar{\theta}] \approx 0, \quad (23)$$

$$\zeta[-ip_\theta - \hat{p}\bar{\theta}] \approx 0, \quad [-i\bar{p}_\theta - \theta\hat{p}]\bar{\zeta} \approx 0 \quad (24)$$

which are the same as the projections on spinors $\zeta, \hat{p}\bar{\zeta}$ of the Grassmannian spinor constraints

$$d_{\theta\alpha} \equiv -ip_{\theta\alpha} - (\hat{p}\bar{\theta})_\alpha - \theta^\beta Z_{\beta\alpha} - Z_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}} \approx 0, \quad (25)$$

$$\bar{d}_{\theta\dot{\alpha}} \equiv -i\bar{p}_{\theta\dot{\alpha}} - (\theta\hat{p})_{\dot{\alpha}} - \bar{Z}_{\dot{\alpha}\beta}\bar{\theta}^{\dot{\beta}} - \theta^\beta Z_{\beta\dot{\alpha}} \approx 0 \quad (26)$$

with quantities $Z_{\alpha\beta}, Z_{\alpha\dot{\beta}}$ defined in (18). From invariance of the variables $\zeta^\alpha, \bar{\zeta}^{\dot{\alpha}}, p_\mu$ under the canonical transformation all bosonic constraints, i.e. $p^2 + m^2 \approx 0$ and $\zeta\hat{p}\bar{\zeta} - j \approx 0$, are not changed. The system with remaining variables and the constraints is described by the above mentioned Lagrangian (17). The Lagrangian (17) reproduces accurately this set of the constraints and nothing else.

Thus we establish that the model described by Lagrangian $L = L_{1/2} + L_{\text{b.s.}}$ is equivalent physically to the model with Lagrangian $L = L_{\text{super}} + L_{\text{b.s.}}$ at classical level. Here $L_{1/2}$ is the Lagrangian (1) of the massive spinning particle (spin 1/2) whereas L_{super} is Lagrangian of the massive $N = 1$ superparticle with tensorial central charges (18)

$$L_{\text{super}} = p\dot{\omega}_\theta + iZ_{\alpha\beta}\theta^\alpha\dot{\theta}^\beta + i\bar{Z}_{\dot{\alpha}\dot{\beta}}\bar{\theta}^{\dot{\alpha}}\dot{\bar{\theta}}^{\dot{\beta}} + iZ_{\alpha\dot{\beta}}(\theta^\alpha\dot{\bar{\theta}}^{\dot{\beta}} - \dot{\theta}^\alpha\bar{\theta}^{\dot{\beta}}) - \frac{e}{2}(p^2 + m^2). \quad (27)$$

Lagrangians $L_{\text{b.s.}}$ of the bosonic spinor in the both equivalent models are quite identical.

It should be noted that the value of constant k in the formula (18) for central charges of the superparticle is nonzero, $k \neq 0$, in the case of the equivalence to the spinning particle. But in general the value $k = 0$ is not forbidden in model of superparticle with central charges. Next we consider the cases both with $k \neq 0$ and $k = 0$. As we see below at $k \neq 0$ and $k = 0$ we have superparticle models with one and two κ -symmetries respectively.

Alternative way for a proof of classical equivalence of the massive spin 1/2 particle (1) and the massive superparticle with central charges (17), at $k \neq 0$, possessing one κ -symmetry is the reduction

of both model to physical degrees of freedom [21]. In the examining positive energy sector after the choice of gauge $\psi_- = \psi_0 - \psi_5 = 0$ for Dirac constraint and exclusion of $\psi_+ = \psi_0 + \psi_5$ by means of the constraint condition we obtain for the physical Grassmannian degrees of freedom of spinning particle [22, 21] the Lagrangian in the form of $L_{1/2, \text{Gr}}^{(\text{ph})} = \frac{i}{2} \vec{\psi} \dot{\vec{\psi}}$. On the other hand the Grassmannian part of the superparticle Lagrangian L_{super} takes the form

$$L_{\text{super}, \text{Gr}}^{(\text{ph})} = i\bar{q}\dot{q} - iq\dot{\bar{q}} + 2k^2 i\eta\dot{\eta}$$

after using of the variables

$$\eta = mr^{-1/2}(\theta\zeta + \bar{\zeta}\bar{\theta}), \quad \sigma = -imr^{-1/2}(\theta\zeta - \bar{\zeta}\bar{\theta}), \quad (28)$$

$$q = r^{-1/2}(\theta\hat{p}\bar{\zeta}), \quad \bar{q} = r^{-1/2}(\zeta\hat{p}\bar{\theta}). \quad (29)$$

Setting

$$q = (\psi_1 + i\psi_2)/2, \quad \bar{q} = (\psi_1 - i\psi_2)/2, \quad \eta = \psi_3/2k$$

we obtain exactly the same Grassmannian part of the Lagrangian

$$L_{\text{super}, \text{Gr}}^{(\text{ph})} = L_{1/2, \text{Gr}}^{(\text{ph})} = \frac{i}{2} \vec{\psi} \dot{\vec{\psi}}. \quad (30)$$

Such Lagrangian for the physical Grassmannian variables comes out also from work [14] in non-Lorentz covariant Grassmannian sector $N = 4 \rightarrow N = 1$ PBGS. In first order formalism the target space action of this work has the Lagrangian

$$L = \vec{P}\vec{\Pi} - P^0\Pi^0 + \frac{e}{2}(P^{02} - \vec{P}^2 - 1) - \Theta\dot{\Theta} - \vec{\Psi}\dot{\vec{\Psi}} \quad (31)$$

where $\Pi^0 = \dot{X}^0 + \Theta\dot{\Theta} + \vec{\Psi}\dot{\vec{\Psi}}$, $\vec{\Pi} = \dot{\vec{Y}} - \dot{\Theta}\vec{\Psi} + \Theta\dot{\vec{\Psi}}$ (we remain here the notations of [14]). In accounting the last expressions, the Lagrangian (31) takes the form

$$L = \vec{P}\dot{\vec{Y}} - P^0\dot{X}^0 + \frac{e}{2}(P^{02} - \vec{P}^2 - 1) - (P^0 + 1) \left[\vec{\Psi} - \frac{1}{P^0 + 1} \vec{P}\Theta \right] \left[\vec{\Psi} - \frac{1}{P^0 + 1} \vec{P}\Theta \right].$$

After using of the variables

$$\vec{\psi} = \sqrt{2}(P^0 + 1)^{1/2} \left[\vec{\Psi} - \frac{1}{P^0 + 1} \vec{P}\Theta \right]$$

we obtain exactly the Lagrangian (30) for Grassmannian variables.

In order to analyse the properties of the obtained massive superparticle with tensorial central charges let us consider the model of spinning particle with index spinor [18, 10, 19] as additional bosonic coordinates. It is naturally because we have used for bosonic spinor the relation $\zeta\hat{p}\bar{\zeta} - j \approx 0$ which is inherent in the index spinor approach. In the Hamiltonian formalism the index spinor sector is restricted by the spinor self-conjugacy conditions

$$d_\zeta \equiv ip_\zeta - \hat{p}\bar{\zeta} \approx 0, \quad \bar{d}_\zeta \equiv -i\hat{p}_\zeta - \zeta\hat{p} \approx 0 \quad (32)$$

which are the second class constraints in the massive case. It is achieved in above model (6) by the substitution $v = -i\hat{p}\bar{\zeta}$, $\bar{v} = i\zeta\hat{p}$. Then $L_{\text{b.s.}}$ (5) takes the form of the index spinor Lagrangian [18]

$$L_{\text{index}} = -i\dot{\zeta}\hat{p}\bar{\zeta} + i\zeta\hat{p}\dot{\bar{\zeta}} - \lambda(\zeta\hat{p}\bar{\zeta} - j). \quad (33)$$

Included in the Lagrangian the constraint $\zeta\hat{p}\bar{\zeta} - j \approx 0$ generates in Hamiltonian formalism the spin constraint

$$\frac{i}{2}(\zeta p_\zeta - \bar{p}_\zeta \bar{\zeta}) - j \approx 0 \quad (34)$$

which together with second class constraints (32) lead [18] to the particle state of the single spin associated with given sector of index spinor. Spin of the particle in the quantum spectrum is the value of the constant j renormalized by ordering constants (thus j can be named “classical spin”).

The realization of the previously considered canonical transformation to the model with Lagrangian $L' = L_{1/2} + L_{\text{index}}$, i.e. L_{index} instead $L_{\text{b.s.}}$ in (6), leads to the Lagrangian

$$\begin{aligned} L' = p\dot{\omega} &+ iZ_{\alpha\beta}\theta^\alpha\dot{\theta}^\beta + i\bar{Z}_{\dot{\alpha}\dot{\beta}}\bar{\theta}^{\dot{\alpha}}\dot{\bar{\theta}}^{\dot{\beta}} + iZ_{\alpha\dot{\beta}}(\theta^\alpha\dot{\bar{\theta}}^{\dot{\beta}} - \dot{\theta}^\alpha\bar{\theta}^{\dot{\beta}}) \\ &+ iY_{\alpha\beta}\zeta^\alpha\dot{\zeta}^\beta + i\bar{Y}_{\dot{\alpha}\dot{\beta}}\bar{\zeta}^{\dot{\alpha}}\dot{\bar{\zeta}}^{\dot{\beta}} + iY_{\alpha\dot{\beta}}(\zeta^\alpha\dot{\bar{\zeta}}^{\dot{\beta}} + \dot{\zeta}^\alpha\bar{\zeta}^{\dot{\beta}}) \\ &- iN(\dot{\zeta}\hat{p}\bar{\zeta} - \zeta\hat{p}\dot{\bar{\zeta}}) \\ &- \frac{e}{2}(p^2 + m^2) - \lambda(\zeta\hat{p}\bar{\zeta} - j). \end{aligned} \quad (35)$$

Here the form $\omega \equiv \dot{\omega} d\tau = dx - id\zeta\sigma\bar{\zeta} + i\zeta\sigma d\bar{\zeta} - id\theta\sigma\bar{\theta} + i\theta\sigma d\bar{\theta}$ is invariant with respect to the transformations of the usual $N = 1$ supersymmetry with Grassmannian spinor parameter and “bosonic

supersymmetry” with c -number spinor parameter [18, 10, 19]. The central charges $Z_{\alpha\beta}$, $Z_{\alpha\dot{\beta}}$ have the same form (18). So the kinetic terms of the space-time coordinate and Grassmannian spinor in L' (35) are identical to the corresponding terms in L (17) and hence the algebras of the fermionic constraints in both models are identical. But the kinetic terms of the index spinor in Lagrangian L' are different from the kinetic terms of the bosonic spinor in Lagrangian L by additional terms with quantities

$$Y_{\alpha\beta} = 2k(k-2)m^2j^{-1}\theta_\alpha\theta_\beta, \quad \bar{Y}_{\dot{\alpha}\dot{\beta}} = -\overline{(Y_{\alpha\beta})},$$

$$Y_{\alpha\dot{\beta}} = -(2k^2 - 4k + 1)m^2j^{-1}\theta_\alpha\bar{\theta}_{\dot{\beta}} \quad (36)$$

which can be regarded as the central charges of the “bosonic SUSY” as well as

$$N \equiv j^{-1} \left[(\theta\hat{p}\bar{\theta}) + 2(2k-1)m^2j^{-1}(\theta\zeta)(\bar{\zeta}\bar{\theta}) \right]. \quad (37)$$

The appearance of these extra terms is the result of modification of index spinor momenta p_ζ , $\bar{p}_{\bar{\zeta}}$ under the canonical transformation and, as consequence, the modification of the spin constraint (34) and bosonic spinor constraints (32) expressed by new variables.

Specific peculiarity of the model (35) with index spinor is an interconnection between usual fermionic supersymmetry and “bosonic one” and at present its meaning is not yet quite clear. Some duality appears in the invariance under permutation of Grassmannian and bosonic spinors both ω -form and certain terms with central charges of different types.

3 Massive superparticle with tensorial central charges. Invariances

The massive superparticle (17) with tensorial central charges possesses the usual target space supersymmetry

$$\delta\theta^\alpha = \epsilon^\alpha, \quad \delta\bar{\theta}^{\dot{\alpha}} = \bar{\epsilon}^{\dot{\alpha}}, \quad \delta x_\mu = i\theta\sigma_\mu\delta\bar{\theta} - i\delta\theta\sigma_\mu\bar{\theta} \quad (38)$$

with constant Grassmannian parameter ϵ^α . As usual in the cases of the formulation without central charge coordinates [23] the Lagrangian L is quasi-invariant. With accounting of the bosonic spinor

equation of motion $\dot{\zeta} = 0$ its variation is the full derivative

$$\delta L = \left(iZ_{\alpha\beta}\epsilon^\alpha\theta^\beta + iZ_{\alpha\dot{\beta}}\epsilon^\alpha\bar{\theta}^{\dot{\beta}} \right)' + \text{c.c.} \quad (39)$$

Then the generators of the supersymmetry transformations

$$\begin{aligned} Q_\alpha &= \frac{\partial}{\partial\theta^\alpha} + (\hat{p}\bar{\theta})_\alpha + \theta^\beta Z_{\beta\alpha} + Z_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}, \\ \bar{Q}_{\dot{\alpha}} &= \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + (\theta\hat{p})_{\dot{\alpha}} + \bar{Z}_{\dot{\alpha}\beta}\bar{\theta}^{\dot{\beta}} + \theta^\beta Z_{\beta\dot{\alpha}} \end{aligned} \quad (40)$$

contain “anomalous” extra piece with central charges (18) [23, 1]. The algebra of SUSY generators

$$\{Q_\alpha, Q_\beta\} = 2Z_{\alpha\beta}, \quad \{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2(p_{\alpha\dot{\beta}} + Z_{\alpha\dot{\beta}}) \quad (41)$$

is the $N = 1$ $D = 4$ SUSY algebra extended by tensorial central charges [1]-[8].

Of course we can introduce the coordinates of central charges introducing terms with derivatives of these coordinates to the multipliers at central charges in (17) [23, 9]. Then the model becomes not only quasi-invariant but SUSY invariant.

The price for the presence of the supersymmetry is the infinite number of the spin states in the spectrum. At the restriction of the bosonic spinor sector to the index spinor one the number of the states in spectrum becomes finite but the supersymmetry disappears. But in both cases, (17) and (35), the models possess local κ -symmetries.

For local transformation of the Grassmannian spinor

$$\delta\theta^\alpha = i\kappa(\bar{\zeta}\tilde{p})^\alpha, \quad \delta\bar{\theta}^{\dot{\alpha}} = -i\bar{\kappa}(\tilde{p}\zeta)^{\dot{\alpha}} \quad (42)$$

and standard Siegel transformation [23, 24] of the space-time coordinate

$$\delta x_\mu = -i\theta\sigma_\mu\delta\bar{\theta} + i\delta\theta\sigma_\mu\bar{\theta} \quad (43)$$

with local complex Grassmannian parameter $\kappa(\tau)$ the variation of the Lagrangians up to a total derivative is

$$\begin{aligned} \delta L &= -2k^2m^2(\theta\zeta + \bar{\zeta}\bar{\theta})(\kappa - \bar{\kappa})' + 2k^2m^2(\theta\zeta + \bar{\zeta}\bar{\theta})'(\kappa - \bar{\kappa}) \\ &\quad - 4km^2j^{-1}[(\theta\hat{p}\bar{\zeta})\zeta\dot{\zeta} + (\zeta\hat{p}\bar{\theta})\bar{\zeta}\dot{\bar{\zeta}}](\kappa - \bar{\kappa}). \end{aligned} \quad (44)$$

As we see, $\delta L = 0$ for real $\kappa = \bar{\kappa}$ at arbitrary values of constant k . But at $k = 0$ we have $\delta L = 0$ for arbitrary complex parameter κ . Thus at $k \neq 0$ when the tensor central charge $Z_{\alpha\beta}$ is present the models have one κ -symmetry with real Grassmannian parameter $\kappa = \bar{\kappa}$. But at $k = 0$ when there is only the vector central charge $Z_{\alpha\dot{\beta}}$ we have two κ -symmetries with complex Grassmannian parameter κ .

A first class constraint is associated to each local invariance in Hamiltonian formalism. As is already noted our systems are described by the fermionic constraints (covariant derivatives) (25), (26). Their Poisson brackets algebra is

$$\begin{aligned} \{d_{\theta\alpha}, d_{\theta\beta}\} &= 2iZ_{\alpha\beta}, \quad \{\bar{d}_{\theta\dot{\alpha}}, \bar{d}_{\theta\dot{\beta}}\} = 2i\bar{Z}_{\dot{\alpha}\dot{\beta}}, \\ \{d_{\theta\alpha}, \bar{d}_{\theta\dot{\beta}}\} &= 2i(p_{\alpha\dot{\beta}} + Z_{\alpha\dot{\beta}}) \end{aligned} \quad (45)$$

with central charges (18). Covariant separation of the fermionic first and second class constraints is achieved by the projection on the spinors $\zeta_\alpha, (\hat{p}\bar{\zeta})_\alpha$. Let us put

$$\chi_\theta \equiv \zeta d_\theta = -i\zeta p_\theta - \zeta \hat{p}\bar{\theta} \approx 0, \quad \bar{\chi}_\theta \equiv \bar{d}_\theta \bar{\zeta} = -i\bar{p}_\theta \bar{\zeta} - \theta \hat{p}\bar{\zeta} \approx 0, \quad (46)$$

$$g_\theta \equiv \bar{\zeta} \tilde{p} d_\theta + \bar{d}_\theta \tilde{p} \zeta = -i(\bar{\zeta} \tilde{p} p_\theta + \bar{p}_\theta \tilde{p} \zeta) - 4k^2 m^2 (\theta \zeta + \bar{\zeta} \bar{\theta}) \approx 0, \quad (47)$$

$$f_\theta \equiv i(\bar{\zeta} \tilde{p} d_\theta - \bar{d}_\theta \tilde{p} \zeta) = \bar{\zeta} \tilde{p} p_\theta - \bar{p}_\theta \tilde{p} \zeta \approx 0. \quad (48)$$

The nonzero Poisson brackets of these projections are

$$\{\chi_\theta, \bar{\chi}_\theta\} = 2ij, \quad \{g_\theta, g_\theta\} = 16k^2 m^2 ij. \quad (49)$$

Thus the constraints $\chi_\theta, \bar{\chi}_\theta$ are always the second class constraints whereas the constraint f_θ is always the first class constraint generating one κ -symmetry with local parameter $(\kappa + \bar{\kappa})$ on variable $(\theta\zeta - \bar{\zeta}\bar{\theta})$, $\{f_\theta, \theta\zeta - \bar{\zeta}\bar{\theta}\} = 2r$, $\delta(\theta\zeta - \bar{\zeta}\bar{\theta}) = ir(\kappa + \bar{\kappa})$. The constraint g_θ is the second class constraint at $k \neq 0$. But at $k = 0$ the constraint g_θ becomes the first class constraint and generates additional κ -symmetry with local parameter $i(\kappa - \bar{\kappa})$ on variable $(\theta\zeta + \bar{\zeta}\bar{\theta})$, $\{g_\theta, \theta\zeta + \bar{\zeta}\bar{\theta}\} = -2ir$, $\delta(\theta\zeta + \bar{\zeta}\bar{\theta}) = ir(\kappa - \bar{\kappa})$.

Thus we obtain the models of the $D = 4$ $N = 1$ massive superparticle with tensorial central charges possessing one or two Siegel κ -symmetries. In the language of the brane theories these models correspond to the BPS superbrane configurations preserving 1/4 or 1/2 of supersymmetry (see [8] and references there).

It should be noted that constant k in the construction of the superparticle appears in the gauge fixing condition under transition from the spinning particle. Therefore at all $k \neq 0$ the superparticle has quite similar systems of the constraints and the same number of physical degrees of freedom. The models at all $k \neq 0$ are equivalent. Under transformations which can be considered as canonical transformations

$$\theta^\alpha \rightarrow \theta^\alpha + br^{-1}(\theta\zeta + \bar{\zeta}\bar{\theta})(\bar{\zeta}\tilde{p})^\alpha, \quad \bar{\theta}^{\dot{\alpha}} \rightarrow \bar{\theta}^{\dot{\alpha}} + br^{-1}(\theta\zeta + \bar{\zeta}\bar{\theta})(\tilde{p}\zeta)^{\dot{\alpha}} \quad (50)$$

where b is real number the Lagrangian L (or L') transforms into the same Lagrangian with ak in place of k where $a \equiv 1 + 2b$. As final result at level of the free superparticle we have two substantially different models of the massive superparticle with tensorial central charges. First of them at $k = 1/\sqrt{2}$ has only tensor central charge $Z_{\alpha\beta}$ and possesses one κ -symmetry. Second model at $k = 0$ has only vector central charge $Z_{\alpha\dot{\beta}}$ and possesses two κ -symmetries.

4 Conclusion

In this work we presented the manifestly Lorenz-invariant formulation of the $D = 4$ $N = 1$ free massive superparticle with tensorial central charges. The model contains a real parameter k and at $k \neq 0$ it has one κ -symmetry while at $k = 0$ the number of κ -symmetries is two.

In process of the construction it is established the equivalence at classical level between the massive $D = 4$ $N = 1$ superparticle with one κ -symmetry and the massive $D = 4$ $n = 1$ spinning particle. But they may lead to distinct quantum theories [21]. Below we establish that the spinning particle and superparticle with tensorial central charges, which have index spinor as additional one, have identical state spectrum. By analogy with results in paper [11, 12, 18] the first operator quantization

of the spinning particle with index spinor described by Lagrangian $L_{1/2} + L_{\text{index}}$ is immediate. Wave function in the model is defined by Dirac spinor with (anti)holomorphic dependence in index spinor of homogeneity degree $2J$ where J is the classical spin j renormalized by the ordering constant. Writing Dirac spinor in terms of Weyl spinors as $\begin{pmatrix} \psi \\ \chi \end{pmatrix}$, in according to analysis carried out in [18] we have in holomorphic case two multispinor fields $\psi_{\alpha_1 \dots \alpha_{2J} \beta}$ and $\chi_{\alpha_1 \dots \alpha_{2J} \dot{\beta}}$ which are symmetrical in $2J$ indices α s. Here β and $\dot{\beta}$ correspond to bispinor index. These field are connected with each other by Dirac equation

$$\begin{pmatrix} 0 & \tilde{p} \\ \hat{p} & 0 \end{pmatrix} \begin{pmatrix} \psi \\ \chi \end{pmatrix} = m \begin{pmatrix} \psi \\ \chi \end{pmatrix} \quad (51)$$

(quantum counterpart of the Dirac constraint (2)). Comparison with superparticle model is more immediate if we take the field $\chi_{\alpha_1 \dots \alpha_{2J} \dot{\beta}}$ as basic one. But the field $\psi_{\alpha_1 \dots \alpha_{2J-1} \alpha_{2J} \beta} = \phi_{(\alpha_1 \dots \alpha_{2J} \beta)} + \phi_{(\alpha_1 \dots \alpha_{2J-1} \epsilon_{\alpha_{2J}}) \beta}$ exhibits simply that two spins $J \pm \frac{1}{2}$ are presented in spectrum at fixed J as it should be when one adds spin J which is given by index spinor and spin $\frac{1}{2}$ which corresponds to the Grassmannian variables ψ_μ, ψ_5 of the pseudoclassical mechanics under quantization.

The quantization of the superparticle (35) is suitable to carry out in variables (28), (29) in term of which the fermionic constraints (46)-(48) takes the extremely simple form

$$\begin{aligned} ip_q + \bar{q} &\approx 0, & i\bar{p}_q + q &\approx 0, \\ ip_\eta + 2k^2\eta &\approx 0, \end{aligned} \quad (52)$$

$$p_\sigma \approx 0.$$

We gauging out the variable σ , the introduce the Dirac brackets for taking into account of the fermionic second class constraints and the represent the remaining fermionic variables q, \bar{q}, η (in fact $\vec{\psi}$) by means of the usual Pauli σ -matrices. Thus the wave function of this problem has two components depending appropriately on index spinor and space-time variables. The quantization of the bosonic spinor sector shows certain difference with [18]. Additional term of the form $q\bar{q}$ in spin constraint (34) arising due to interaction of bosonic and fermionic sectors leads to different homogeneity degrees (which correspond

to different representations of Lorentz group) for two components of wave function. Bosonic spinor constraints (32) ((anti)homogeneity conditions) acquire the additional terms both with $q\bar{q}$ and also $q\eta$ (or $\bar{q}\eta$). These last terms, which are proportional σ_+ (or σ_-), $\sigma_{\pm} \equiv (\sigma_1 \pm i\sigma_2)/2$ in matrix realization of Grassmannian variables, connect two components of wave function. As result the irreducible $(2J+1)$ -component spinor field $\phi_{\alpha_1 \dots \alpha_{2J+1}}$, in term of which one component of wave function is determined, is expressed by Dirac equation

$$p_{\gamma\dot{\beta}}\chi_{\alpha_1 \dots \alpha_{2J}}^{\dot{\beta}} = m\phi_{\alpha_1 \dots \alpha_{2J}\gamma}. \quad (53)$$

via field $\chi_{\alpha_1 \dots \alpha_{2J}}^{\dot{\beta}}$ which determines second component of wave function. This last field $\chi_{\alpha_1 \dots \alpha_{2J}}^{\dot{\beta}}$ can be identified with basic field of the spinning particle spectrum.

In case of models (6) and (17), when there is not present the truncation of bosonic spinor sector to the index one because of absence of bosonic spinor constraints, the quantum equivalence apparently remains too. One can expect it from the quite identity of bosonic sectors of the models (6) and (17) and identifying of physical fermionic degrees of freedom which has been demonstrated in Sec. 2.

In case of the Lagrangian (17) one can include vector central charge Z_{μ} into vector of space-time momentum by the shift $p_{\mu} \rightarrow p_{\mu} + Z_{\mu}$ after taking into account the bosonic spinor equation of motion $\dot{\zeta} = 0$. Therefore at $k = 0$, when there is vector central charge only, it disappears completely from the action and superparticle model reduces in fact to massless case. Unlike this in the particle model (35) with index bosonic spinor at $k = 0$ the redefinition of momentum does not exclude vector central charge due to accompanying modification of bosonic spinor and spin constraints. In this case the wave function contains two usual spin-tensor fields $\phi_{\alpha_1 \dots \alpha_{2J\pm 1}}$, satisfying massive Klein-Gordon equation and disconnected with each other because of missing terms with $q\eta$ in bosonic spinor constraints.

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